**Математика. Текст 1.**

**Harmonic Subtangent Structures**

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Copyright © 2014 Adara M. Blaga. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. The concept of harmonic subtangent structures on almost subtangent metric manifolds is introduced and a Bochner-type formula is proved for this case. Conditions for a subtangent harmonic structure to be preserved by harmonic maps are also given.

**1. Introduction**

Inspired by the paper of Jianming [1], we introduce the

notion of *harmonic almost subtangent structure* and underline

the connection between harmonic subtangent structures and

harmonic maps. It is well known that harmonic maps play

an important role in many areas of mathematics. They

often appear in nonlinear theories because of the nonlinear

nature of the corresponding partial differential equations.

In theoretical physics, harmonic maps are also known as

sigma models. Remark also that harmonic maps between

manifolds endowed with different geometrical structures

have been studied in many contexts: Ianus and Pastore

treated the case of contact metric manifolds [2], Bejan

and Benyounes the almost para-Hermitian manifolds [3],

Sahin the locally conformal KЁahler manifolds [4], Ianus

et al. the quaternionic KЁahler manifolds [5], Jaiswal the

Sasakian manifolds [6], Fetcu the complex Sasakian manifolds

[7], Li the Finsler manifolds [8], and so forth. Fotiadis

studied the noncompact case, describing the problem of

finding a harmonic map between noncompact manifolds

[9].

Let 𝑀 be a smooth, 𝑚-dimensional real manifold for

which we denote by 𝐶∞(𝑀) the real algebra of smooth real

functions on𝑀, by Γ(𝑇𝑀) the Lie algebra of vector fields on

𝑀, andby𝑇𝑟

𝑠

(𝑀) the𝐶∞(𝑀)-moduleof tensor fieldsof (𝑟, 𝑠)-

type on 𝑀. An element of 𝑇1

1

(𝑀) is usually called *vector* 1*-*

*form* or *affinor*.

Recall the concept of almost tangent geometry. *Definition 1* (see [10]). 𝐽 ∈ 𝑇1

1

(𝑀) is called almost tangent

structure on𝑀if it has a constant rank and

Im𝐽 = ker 𝐽. (1)

The pair (𝑀, 𝐽) is called *almost tangent manifold*.

The name is motivated by the fact that (1) implies the

nilpotence 𝐽2 = 0 exactly as the natural tangent structure of

tangent bundles. Denoting rank𝐽 = 𝑛 it results in 𝑚 = 2𝑛. If

in addition, we assume that 𝐽 is integrable, that is,

(2)

then 𝐽 is called *tangent structure* and (𝑀, 𝐽) is called *tangent*

*manifold*.

From [11] we deduce some aspects of tangent manifolds:

(i) the distribution Im𝐽(= ker 𝐽) defines a foliation;

(ii) there exist local coordinates (𝑥, 𝑦) = (𝑥𝑖, 𝑦𝑖)

1≤𝑖≤𝑛 on

𝑀such that 𝐽 = 𝜕/𝜕𝑦 𝑖

⊗ 𝑑𝑥 𝑖

; that is,

. (3We call (𝑥, 𝑦) *canonical coordinates* and the change of canonical

coordinates (𝑥, 𝑦) → (̃𝑥, ̃ 𝑦) is given by

So another description can be obtained in terms of 𝐺-

structures. Namely, a tangent structure is a 𝐺-structure with

[12]

and 𝐺 is the invariance group of matrix 𝐽 = (𝑂

𝑛

that is,

𝐶 ∈ 𝐺 if and only if 𝐶 ⋅ 𝐽 = 𝐽 ⋅ 𝐶.

The natural almost tangent structure 𝐽 of𝑀 = 𝑇𝑁is an

example of tangent structure having exactly the expression (3)

if (𝑥𝑖) are the coordinates on 𝑁 and (𝑦𝑖) are the coordinates

in the fibers of 𝑇𝑁 → 𝑁. A class of examples is obtained by

duality [12]: if 𝐽 is an (integrable) endomorphism with 𝐽2 = 0,

then its dual 𝐽∗ : Γ(𝑇∗𝑀) → Γ(𝑇 ∗ 𝑀), given by 𝐽∗𝛼 := 𝛼 ∘ 𝐽

for 𝛼 ∈ Γ(𝑇∗𝑀), is (integrable) endomorphism with (𝐽∗)2 =

0.

If the condition in the Definition 1 is weakened, requiring

that only 𝐽 squares to 0, we call 𝐽 *almost subtangent structure*.

In this case, Im𝐽 ⊂ ker 𝐽.

**Математика. Текст 2.**

**Novel Properties of Fuzzy Labeling Graphs**

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Copyright © 2014 A. Nagoor Gani et al.This is an open access article distributed under the Creative CommonsAttribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. Theconcepts of fuzzy labeling and fuzzy magic labeling graph are introduced. Fuzzy magic labeling for some graphs like path, cycle, and star graph is defined. It is proved that every fuzzy magic graph is a fuzzy labeling graph, but the converse is not true.We have shown that the removal of a fuzzy bridge from a fuzzy magic cycle with odd nodes reduces the strength of a fuzzy magic cycle.

Some properties related to fuzzy bridge and fuzzy cut node have also been discussed.

**1. Introduction**

Fuzzy set is a newly emerging mathematical framework

to exemplify the phenomenon of uncertainty in real life

tribulations. It was introduced by Zadeh in 1965, and the

concepts were pioneered by various independent researches,

namely, Rosenfeld [1] and Bhutani and Battou [2] during

1970s. Bhattacharya has established the connectivity concepts

between fuzzy cut nodes and fuzzy bridges entitled “*Some*

*remarks on fuzzy graphs* [3].” Several fuzzy analogs of graph

theoretic concepts such as paths, cycles, and connectedness

were explored by them.There are many problems, which can

be solved with the help of the fuzzy graphs.

Though it is very young, it has been growing fast and

has numerous applications in various fields. Further, research

on fuzzy graphs has been witnessing an exponential growth,

both within mathematics and in its applications in science

and Technology. A fuzzy graph is the generalization of the

crisp graph. Therefore it is natural that many properties are

similar to crisp graph and also it deviates at many places.

In crisp graph, a bijection : 𝑉 ∪ 𝐸 → 𝑁 that

assigns to each vertex and/or edge if 𝐺 = (𝑉, 𝐸), a unique

natural number is called a labeling. The concept of magic

labeling in crisp graph was motivated by the notion of magic

squares in number theory. The notion of magic graph was

first introduced by Sunitha and Vijaya Kumar [4] in 1964.He

defined a graph to be magic if it has an edge-labeling, within

the range of real numbers, such that the sum of the labels

around any vertex equals some constant, independent of the

choice of vertex. This labeling has been studied by Stewart

[5, 6] who called the labeling as super magic if the labels

are consecutive integers, starting from 1. Several others have

studied this labeling.

**Математика. Текст 3.**

**Startpoints and** (𝛼, 𝛾)**-Contractions in**

**Quasi-Pseudometric Spaces**

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We introduce the concept of *startpoint* and *endpoint* for multivalued maps defined on a quasi-pseudometric space.We investigate

the relation between these new concepts and the existence of fixed points for these set valued maps.

*Dedicated to my beloved Clґemence on the occasion of her 25th birthday*

**1. Introduction**

In the last few years there has been a growing interest in the

theory of quasi-metric spaces and other related structures

such as quasi-normed cones and asymmetric normed linear

spaces (see, e.g., [1]), because such a theory provides an

important tool and a convenient framework in the study

of several problems in theoretical computer science, applied

physics, approximation theory, and convex analysis. Many

works on general topology have been done in order to extend

the well-known results of the classical theory. In particular,

various types of completeness are studied in [2], showing,

for instance, that the classical concept of *Cauchy sequences*

can be accordingly modified. In the same reference, which

uses an approach based on uniformities, the *bicompletion*

of a 𝑇

0-quasi-pseudometric has been explored. It is worth

mentioning that, in the fixed point theory, *completeness* is a

key element, since most of the constructed sequences will be

assumed to have a *Cauchy type* property.

It is the aim of this paper to continue the study of quasipseudometric

spaces by proving some fixed point results

and investigating a bit more the behaviour of set-valued

mappings. Thus, in Section 3 a suitable notion of (𝛼, 𝛾)-

contractive mapping is given for self-mappings defined on

quasi-pseudometric spaces and some fixed point results are

dis *endpoint* for set-valued mappings are introduced and different

variants of suchconcepts, aswell as their connectionswith

the fixed point of a multivalued map, are characterized.

For recent results in the theory of asymmetric spaces, the

reader is referred to [3–8].

**2. Preliminaries**

*Definition 1.* Let 𝑋 be a nonempty set. A function : 𝑋 ×

𝑋 → [0,∞) is called a *quasi-pseudometric* on 𝑋 if

…for all 𝑥, 𝑦, 𝑧 ∈ 𝑋. Moreover, if (𝑥,𝑦) = 0 = 𝑑(𝑦,𝑥)

⇒ 𝑥 = 𝑦, then 𝑑 is said to

be a 𝑇0-*quasi-pseudometric.* Thelatter condition is referred to

as the 𝑇 0-condition.

*Remark 2.* (i) Let 𝑑 be a quasi-pseudometric on 𝑋; then the

map 𝑑−1 defined by 𝑑−1(𝑥, 𝑦) = 𝑑(𝑦, 𝑥) whenever 𝑥, 𝑦 ∈ 𝑋 is

also a quasi-pseudometric on 𝑋, called the *conjugate* of 𝑑. In

the literature, 𝑑−1 is also denoted by 𝑑𝑡 or 𝑑.

(ii) It is easy to verify that the function 𝑑𝑠 defined by 𝑑 :=

𝑑 ∨ 𝑑−1, that is, 𝑑𝑠(𝑥, 𝑦) = max{𝑑(𝑥, 𝑦), 𝑑(𝑦, 𝑥)}, defines a

metric on 𝑋 whenever 𝑑 is a 𝑇

0-quasi-pseudometric on 𝑋.cussed. In Sections 4 and 5, the notions of *startpoint* and

Let (𝑋, 𝑑) be a quasi-pseudometric space. For 𝑥 ∈ 𝑋 and

….. (1)

denotes the open 𝜀-ball at 𝑥. The collection of all such balls

yields a base for the topology 𝜏(𝑑) induced by 𝑑 on 𝑋. Hence,

for any 𝐴 ∈ 𝑋, we will, respectively, denote by int𝜏(𝑑)

𝐴 and cl𝜏(𝑑)

𝐴 the interior and the closure of the set 𝐴 with respect

to the topology 𝜏(𝑑).

Similarly, for 𝑥 ∈ 𝑋 and 𝜀 ≥ 0,

(2)

denotes the closed 𝜀-ball at 𝑥. We will say that a subset𝐸 ⊂ 𝑋

is *join-closed* if it is 𝜏(𝑑𝑠)-closed, that is, closed with respect to

the topology generated by 𝑑𝑠.The topology 𝜏(𝑑𝑠) is finer than

the topologies 𝜏(𝑑) and 𝜏(𝑑−1).

**Математика. Текст 4.**

**Common Fixed Point Theorems of Multivalued Maps in**

**Fuzzy Ultrametric Spaces**

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Copyright © 2013 A. F. Sayed. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. In the setting of fuzzy ultrametric spaces, we study common fixed point theorems of multivalued maps. Our results unify, extend, and generalize some related common fixed point theorems of the literature for both ultrametric spaces (Wang and Song (2013),

Gajiґc (2002) and (2001)) and fuzzy metric spaces (Vijayaraju and Sajath (2011)).

**1. Introduction**

In 1965, Zadeh [1] introduced the theory of fuzzy sets. Many

authors introduced the notion of fuzzy metric space in different

ways. George and Veeramani [2] modified the concept

of fuzzy metric space introduced by Kramosil and Michalek

[3] and defined Hausdorff topology in fuzzy metric space.

Several authors [4–10] studied and developed the concept

in different directions and proved fixed point theorems in

fuzzy metric spaces. Vijayaraju and Sajath [11] extended

some previous results and proved some common fixed points

theorems for hybrid pair of single and multivalued maps

under hybrid contractive conditions. Wang and Song [12]

established some results on coincidence and common fixed

point for two pairs of multivalued and single-valued maps

in ultrametric spaces. In 2009, Savchenko and Zarichnyi [13]

introduced the concept of fuzzy ultrametric space. Sedghi

and Shobe [14] proved common fixed point theorems for

self-maps satisfying contractive conditions on spherically

complete fuzzy ultrametric spaces. In this paper, in the setting

of fuzzy ultrametric spaces, we study common fixed point

theorems ofmultivaluedmaps. Our results unify, extend, and

generalize some related common fixed point theorems of the

literature for both ultrametric spaces [12, 15, 16] and fuzzy

metric spaces [11].

**2. Preliminaries and Notations**

*Definition 1* (see [1]). Let 𝑋 be any nonempty set. A fuzzy

set 𝐴 in 𝑋 is a function with domain 𝑋 and values in [0, 1].

*Definition 2* (see [17]). A binary operation ∗ : [0,1] 2

→

[0, 1] is called a continuous triangular norm(shortly *t*-norm)

if it satisfies the following conditions:

(1) ∗ is associative and commutative,

(2) ∗ is continuous,

(3) 𝑎 ∗ 1 = 𝑎 for all 𝑎, 𝑏, 𝑐, 𝑑 ∈ [0, 1],

(4) 𝑎 ∗ 𝑏 ≤ 𝑐 ∗ 𝑑 whenever 𝑎 ≤ 𝑐 and 𝑏 ≤ 𝑑 for all

𝑎, 𝑏, 𝑐, 𝑑 ∈ [0, 1].

*Definition 3* (see [2]). The 3-tuple (*X*, *M*, ∗) is called a fuzzy

metric space if 𝑋 is an arbitrary (nonempty) set, ∗ is a

continuous *t*-norm, and *M* is a fuzzy set on 𝑋

2

× [0,∞)

satisfying the following conditions, for all 𝑥, 𝑦, 𝑧 ∈ 𝑋 and

each 𝑡 and 𝑠 > 0:

(1) (𝑥, 𝑦, 𝑡) = 0,

(2) (𝑥, 𝑦, 𝑡) = 1 if and only if 𝑥 = 𝑦,

(3) (𝑥, 𝑦, 𝑡) = 𝑀(𝑦, 𝑥, 𝑡),

(4) (𝑥, 𝑦, 𝑡) ∗𝑀(𝑦, 𝑧, 𝑡) ≤ 𝑀(𝑥, 𝑧, 𝑡 + 𝑠),

(5) (𝑥, 𝑦, ⋅) : (0,∞) → [0, 1] is continuous.

𝑀 is called a fuzzy metric on 𝑋.The functions 𝑀(𝑥, 𝑦, 𝑡)

denote the degree of nearness between *x* and *y* with respect

to 𝑡, respectively. ball 𝐵(𝑥, 𝑟, 𝑡) with center 𝑥 ∈ 𝑋 and radius 0 < 𝑟 < 1 is

defined by

𝐵 (𝑥, 𝑟, 𝑡) = {𝑦 ∈ 𝑋 :𝑀(𝑥,𝑦, 𝑡) > 1 − 𝑟} . (1)

A subset 𝐴 ⊂ 𝑋 is called open if, for each 𝑥 ∈ 𝐴, there

exist 𝑡 > 0 and0 < 𝑟 < 1such that 𝐵(𝑥, 𝑟, 𝑡) ⊂ 𝐴. Let 𝜏 denote

the family of all open subsets of 𝑋.Then 𝜏 is a topology on 𝑋

induced by the fuzzy metric 𝑀. This topology is Hausdorff

and first countable.

*Definition 4* (see [18]). Let (𝑋, 𝑑) be a metric space. If the

metric 𝑑 satisfies strong triangle inequality

𝑑 (𝑥, 𝑦) ≤ max {𝑑 (𝑥, 𝑧) , 𝑑 (𝑧, 𝑦)}, ∀𝑥,𝑦,𝑧∈𝑋, (2)

then 𝑑 is called an ultrametric on 𝑋 and the pair (𝑋, 𝑑) is

called an ultrametric space.

*Definition 5* (see [18]). An ultrametric space (𝑋, 𝑑) is said to

be spherically complete if every shrinking collection of balls

in 𝑋 has a nonempty intersection.